

Anomalies and Deconfinement ^{*}

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Abstract

I discuss how instanton effects can be wiped-out due to the existence of anomalies. I first consider Compact Quantum Electrodynamics in 3 dimensions where confinement of electric charge is destroyed when fermions are added so that a Chern-Simons term is generated as a one-loop effect. I also show that a similar phenomenon occurs in the two-dimensional abelian chiral Higgs model. In both cases anomalies (parity anomaly, gauge anomaly) are responsible of the deconfinement mechanism.

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1 Introduction

Compact Quantum Electrodynamics in $d = 2 + 1$ dimensions ($CQED_3$) is a nice example on how non-perturbative effects can lead to the confinement of a fundamental quantum number such as electric charge.

Consider for example a $d = (3 + 1)$ $SO(3)$ gauge theory with spontaneous symmetry breaking through a Higgs field in the adjoint. As it is well-known, the resulting classical equations of motion have regular *static* solutions: the well-honored 't Hooft-Polyakov monopoles [1]-[2]. These static solutions in $3 + 1$ dimensions can be taken as instanton solutions in 3-dimensional Euclidean space-time. One can then analyse the effects of instantons in the dimensionally reduced theory, 3-dimensional Compact Quantum Electrodynamics (After symmetry breaking of the original (compact) $SO(3)$ group, one ends with a residual compact abelian ($U(1)$) symmetry ensuring the existence of *regular* solutions with finite action).

In $d = 2 + 1$ the Coulomb potential for a $U(1)$ gauge theory is, at the classical level, logarithmic. In the case of $CQED_3$, Polyakov [3] showed, fifteen years ago that the interaction between electric charges becomes linear due to instanton (monopole) effects.

In fact, Polyakov [3] observed that, at the one-loop level, the theory is equivalent to a Coulomb gas which exhibits Debye screening of magnetic (monopole) charges for arbitrarily weak coupling. He then showed how this screening of magnetic charge implies confinement of electric charge: due to the monopole background, an electric string between two fixed charges is stabilized. The length scale of the crossover from logarithmic to linear potential becomes exponentially large at weak coupling.

What happens with confinement when a Chern-Simons (CS) term is added to this $d = 3$ model? I shall address to this question in the first part of my talk but, before advancing the answer, let me discuss why this is an interesting question.

Originally [4] the Chern-Simons term was introduced in $d = 3$ gauge theories as a way of producing symmetry breaking without use of the Higgs fields: due to the presence of the Chern-Simons term, the originally massless QED_3 photon becomes massive. For this reason the model is known as Topologically massive QED_3 .

Another important property of a gauge theory with Chern-Simons term can be understood by analysing the classical equations of motion. Calling Q

the electric charge and Φ the magnetic flux, one of the equations of motion leads to a basic relation:

$$Q = \mu\Phi \tag{1}$$

where μ is the coefficient of the Chern-Simons term. If one then considers a tube with magnetic flux (a Nielsen-Olesen vortex [5]) it necessarily carries electric charge [6]. Moreover, in the non-abelian case μ needs to be quantized (due to gauge invariance requirements [4]) and then both flux and charge of vortices are quantized [7]. One is then dealing with fractional statistic objects a fact that attracted people from Condensed Matter physics to the subject [8].

Another reason to include a Chern-Simons term when studying QED_3 or QCD_3 is the following: in $d = 3$, integration over fermions leads to a Chern-Simons term as a one-loop effect related to violation of parity invariance [9]. In the same way a photon mass arises in QED_2 (Schwinger mechanism) as a result of the impossibility of regularizing the fermion determinant both in a gauge invariant and a chiral invariant way, the impossibility of regularizing the $d = 3$ fermion determinant both respecting gauge invariance and parity invariance produces a Chern-Simons term as a one-loop effect. Then, in 3 dimensions, if one is going to study a gauge theory with charged fermions, the CS term is there.

As I mentioned above, in the presence of a CS term the photon becomes massive so that long-range correlations, necessary for Debye screening of the monopole charged gas, disappear. One should then expect that monopoles and anti-monopoles do not respond anymore as a charged plasma but as a gas of "molecules" or "dipoles" which does not exhibit Debye screening. If this were true, the Wilson loop for external heavy charges would no longer have an area law and confinement should be destroyed. In particular, such a result should be welcome in models intended to describe the physics of high temperature superconductors where spinless charge carriers and neutral spin carriers should be *separatedly* part of the physical spectrum (This being impossible in a confined phase).

The deconfinement scenario was discussed by several authors [10]- [11]. I will describe in this talk the approach we developed in collaboration with Eduardo Fradkin [12]. From our treatment, it will be clear that the lack of gauge invariance of the fermionic determinant in a monopole background is

at the origin of deconfinement of electric charge. (Remember that the Chern-Simons term arises as a result of violation of parity invariance when computing the fermion determinant in a gauge-invariant way. This means that the resulting CS term is necessarily gauge-invariant under *topologically trivial* gauge transformations or in topologically trivial gauge field backgrounds but not when topology enters in the game).

That instanton effects might be wiped out by anomalies can be tested in other simpler models. In particular, I will analyse in the second part of my talk a two-dimensional model (the abelian chiral Higgs model) in which the coupling to Weyl fermions produce a (gauge) anomaly. Again instanton effects disappear, as we have shown in collaboration with Marta Trobo [13].

2 Confinement in $CQED_3$

Let me start by describing how Polyakov [3] proved confinement of electric charge in $CQED_3$ without CS term. Consider the $SO(3)$ Georgi-Glashow model in d=3 euclidean dimensions with Lagrangian:

$$L = \frac{1}{4} \vec{F}_{\mu\nu}^2 + \frac{1}{2} D_\mu \vec{\phi}^2 + V[\vec{\phi}^2], \quad (2)$$

$$D_\mu \vec{\phi} = \partial_\mu \vec{\phi} + e \vec{A}_\mu \wedge \vec{\phi} \quad (3)$$

and $V[\vec{\phi}^2]$ a symmetry breaking potential taking its minimum at $\vec{\phi} = \vec{\phi}_0$. The Euler-Lagrange equations arising from (2) have regular solutions with finite action which are just the static 't Hooft-Polyakov [1]-[2] monopole solutions of the corresponding (3+1) model. Although its exact form is not important here, let us indicate that if we define the electromagnetic tensor $\mathcal{F}_{\mu\nu}$ associated with the residual $U(1)$ symmetry so that:

$$\lim_{|x| \rightarrow \infty} \mathcal{F}_{\mu\nu} = \vec{\phi} \cdot \vec{F}_{\mu\nu}, \quad (4)$$

then the magnetic field B_μ^{mon} for a monopole located at $\vec{x} = \vec{R}$ is given by:

$$B_\mu^{mon}(\vec{x}, \vec{R}) = \frac{1}{2} \epsilon^{\mu\nu\alpha} \mathcal{F}_{\nu\alpha}^{mon} \quad (5)$$

so that at large distances,

$$\lim_{x \rightarrow \infty} B_\mu^{mon}(\vec{x}, \vec{R}) \sim \frac{1}{2} \frac{x_\mu - R_\mu}{|\vec{x} - \vec{R}|^3} \quad (6)$$

These monopole solutions have finite action and can be taken as instantons in a non-perturbative analysis of $CQED_3$. To this end, consider the partition function for the model with dynamics described by Lagrangian (2):

$$\int D\vec{A}_\mu D\vec{\phi} \exp(-S[\vec{A}_\mu, \vec{\phi}]) \quad (7)$$

with

$$S[\vec{A}_\mu, \vec{\phi}] = \int d^3x L \quad (8)$$

Let us now perform a semiclassical expansion by first considering small fluctuations around a charge-1 monopole solution:

$$A_\mu = A_\mu^{mon}(\vec{x}, \vec{R}) + a_\mu \quad (9)$$

$$\phi_\mu = \phi_\mu^{mon}(\vec{x}, \vec{R}) + \varphi \quad (10)$$

or, calling $F \equiv (A_\mu, \phi)$, $f \equiv (a_\mu, \varphi)$,

$$F = F^{mon}(\vec{x}, \vec{R}) + f. \quad (11)$$

One then has for action (8) up to second order in fluctuations:

$$\begin{aligned} S[A_\mu, \phi] &= S[A_\mu^{mon}(\vec{x}, \vec{R}), \phi^{mon}(\vec{x}, \vec{R})] + \underbrace{\int dx f(x) \frac{\delta S}{\delta F(x)}}_{=0} \Big|_{F^{mon}} + \\ &\int dxdy f(x) \frac{\delta^2 S}{\delta F(x) \delta F(y)} \Big|_{F^{mon}} f(y) \end{aligned} \quad (12)$$

or:

$$S[A_\mu, \phi] = S^{mon} + \int d^3x d^3y f(x) S^{II}(x, y) f(y) \quad (13)$$

One has to be carefull in using (13) due to the existence of zero-modes related to invariances of the classical theory. In particular, associated with

translation invariance there is a direction in which the integral over Bessel-Fourier coefficients c_n :

$$F = F^{mon} + \sum_n c_n f_n \quad (14)$$

is not gaussian. (In (14) f_n are the eigenfunctions of the quadratic form in S^{II}). One cannot then just write for the partition function measure:

$$DF = \prod_n dc_n \quad (15)$$

since then Z becomes infinite. Instead, one eliminates from the sum in eq.(14) (the product in (15)) the coefficient accompanying the zero-mode, trading it by the collective coordinate \vec{R} fixing the position of the monopole. One then has instead of (15):

$$DF = Nd\vec{R} \prod' dc_n \quad (16)$$

where the prime indicates that the zero-mode contribution has been eliminated from the product and N is a normalization constant. In a completely analogous way one handles the problem of gauge zero-modes. Once this is done, one is left with gaussian integrations leading to:

$$Z^{(1)} = N \int D\vec{R} \exp[-S^{mon}] \det^{-\frac{1}{2}} S^{II} \det^{\frac{1}{2}} \Delta_{FP} \quad (17)$$

with the superscript (1) indicating the charge-1 monopole contribution to the partition function and Δ_{FP} the Faddeev-Popov operator.

In order to compute the contribution to Z coming from arbitrarily charged monopoles (i.e., to include all topological sectors), we shall consider, following ref.[3], a superposition of N widely separated monopoles of charge ± 1 leading to a charge n configuration. The radius of each ± 1 monopole is of the order of the inverse of the vector meson mass $M_W \sim e\phi_0$ so that if we call \vec{R}_a the position of the a -th ± 1 monopole, $a = 1, 2, \dots N$, widely separated means:

$$R_{ab} \equiv |\vec{R}_a - \vec{R}_b| \gg \frac{1}{M_W} \quad (18)$$

Performing an expansion as that in eq.(12) in each monopole sector one arrives to:

$$S = S^{mon(N)} + S^{II} \quad (19)$$

with:

$$S^{mon(N)} = \sum_{n_a=\pm 1} n_a^2 S^{mon} + \frac{2\pi}{e^2} \sum_{n_a \neq n_b} \frac{n_a n_b}{R_{ab}} + O\left(\frac{1}{M_W R_{ab}}\right) \quad (20)$$

the action for an N monopole superposition.

With this, one can compute the contribution of all topological sectors to the partition function. The answer is:

$$Z = \sum_{N, \{n_a\}} \int \prod_a d\vec{R}_a \frac{\xi^N}{N!} \exp\left[-\frac{2\pi}{e^2} \sum_{n_a \neq n_b} \frac{n_a n_b}{R_{ab}}\right] \quad (21)$$

where $\prod_a d\vec{R}_a$ is the integration measure over all monopole locations in a given superposition,

$$\xi = M_W^{\frac{7}{2}} \exp[-S^{mon}] \det^{-\frac{1}{2}} S^{II} \det^{\frac{1}{2}} \Delta_{FP} \quad (22)$$

and $\{n_a\}$ represents different superpositions of charge ± 1 monopoles leading to a charge n configuration. Accordingly, the $N!$ has been included in order to avoid double counting. Here we have used the factorization of determinants in an N -monopole background into the product of determinants in a ± 1 monopole background (valid whenever condition (20) holds).

Now Z as given by eq.(21) coincides with the partition function for a Coulomb gas of *magnetically* charged particles (with charge $n_a = \pm 1$) interacting through a $\frac{1}{R_{ab}}$ potential. This gas exhibits Debye screening of magnetic charge, this being in turn responsible for confinement of electric charge. Indeed, as shown by Polyakov [3] the Wilson loop computed from the model with partition function (21) exhibits an area law behavior:

$$\lim_{T \rightarrow \infty} \langle \exp[i \oint A_\mu^3 dx^\mu] \rangle \sim \exp[-E(R)T] \quad (23)$$

$$E(R) = \gamma R \quad (24)$$

with R the distance between two external electric test-charges and γ a constant calculable in terms of ξ as given by (22).

3 Adding a Chern-Simons term

Either one adds (massive) fermions or a Chern-Simons term to the Lagrangian (2) the confinement scenario described above is radically changed.

To see this, let us first remind that, given the fermionic Lagrangian in $d = 3$ Euclidean dimensions,

$$L_F = \bar{\psi}(i \not{\partial} + e \not{A} + im)\psi \equiv \bar{\psi}D[A]\psi \quad (25)$$

the associated fermion determinant up to one-loop takes the form [9]:

$$\log \det D[A] = \frac{m}{|m|} \frac{ie^2}{8\pi^2} S_{CS} + \frac{1}{4\pi|m|} \int d^3x F_{\mu\nu}^2, \quad (26)$$

where S_{CS} is the Chern-Simons action:

$$S_{CS} = \text{tr} \int d^3x \epsilon^{\mu\nu\alpha} (A_\mu \partial_\nu A_\alpha + \frac{2}{3} e A_\mu A_\nu A_\alpha) \quad (27)$$

As mentioned before, the emergency of this parity non-conserving term is due to the impossibility of regularizing the fermionic determinant both respecting gauge and space-time reflection invariances [9]. As it is well-known, the CS term is topological in the sense it does not depend on the metric. Then, either in Minkowski or Euclidean space it appears with an i factor in the total effective action resulting from integrating out fermions:

$$S_{eff} = S[\vec{A}_\mu, \vec{\phi}] + i\mu S_{CS} \quad (28)$$

with $S[\vec{A}_\mu, \vec{\phi}]$ as defined in (8) and $\mu = \frac{e^2}{8\pi}$.

Solutions to the equations of motion associated with action (28) are complex and, what is worse, they lead to an infinite action [10]. Leaving aside these solutions, which are useless in a non-perturbative calculation, we shall study the effect of adding the Chern-Simons term when the *old* monopole solutions are taken as instantons. This is the natural thing to do if one considers the CS term as a one-loop effect arising from integration of fermions. Now, although $S_{CS}|_{A_\mu^{mon}} = 0$, one has:

$$\frac{\delta S_{eff}}{\delta A_\mu} \Big|_{A_\mu^{mon}} \neq 0, \quad (29)$$

then, repeating the calculation in Section 2 in order to integrate quadratic fluctuations one get, appart from the classic and quadratic terms already present in (17), a linear term:

$$S_{eff} = S^{mon} + \frac{ie^2}{\pi} \int d^3x B_\mu^{mon} a^\mu + S^{II}, \quad (30)$$

One can easily eliminate this linear term but then the classical term is modified:

$$S_{eff} = S^{mon} + \frac{e^4}{\pi^2} \int d^3x d^3y B_\mu^{mon}(x) S^{(2)\mu\nu}(x, y) B_\nu^{mon}(y) + \bar{S}^{II}. \quad (31)$$

where the actual form of $S^{(2)\mu\nu}(x, y)$ is not important here

Again, one considers a monopole superposition as before, performs the change of variables (11), separates out collective coordinates \vec{R}_a , etc. The new term in (31) gives an extra contribution arising from the magnetic monopole field:

$$\lim_{x \rightarrow \infty} \vec{B}^{mon}(\vec{x}, \vec{R}) \sim \frac{1}{2} \sum_a n_a \frac{\vec{x} - \vec{R}_a}{|\vec{x} - \vec{R}_a|^3} \quad (32)$$

I will skip details and just quote from Ref.[12] the relevant contribution coming from the new term in (31):

$$\frac{e^4}{\pi^2} \int d^3x d^3y B_\mu^{mon}(x) S^{(2)\mu\nu}(x, y) B_\nu^{mon}(y) = -\frac{e^2}{16\pi^2} \sum_{n_a, n_b} n_a n_b R_{ab} \quad (33)$$

so that, instead of partition function (21) one now gets:

$$Z = \sum_{N, \{n_a\}} \int \prod_a d\vec{R}_a \frac{\xi^N}{N!} \exp\left[-\frac{2\pi}{e^2} \sum_{n_a \neq n_b} \frac{n_a n_b}{R_{ab}} + \frac{e^2}{16\pi^2} \sum_{n_a, n_b} n_a n_b R_{ab}\right] \quad (34)$$

The presence of the linear term in (34) implies that confinement is destroyed. Monopoles and antimonopoles themselves become confined due to a linear potential, forming a gas of molecules instead of a charged plasma. Debye screening is lost and the linear potential between electric charges disappears.

There is an alternative way to see that monopole contribution is wiped out from the partition function without resource of non-perturbative calculations. I shall describe it in the next section.

4 Integrating over all field configurations

Let us consider for simplicity an abelian gauge theory although the $SO(3)$ case can be identically treated. The partition function for our model is:

$$Z = \int DA_\mu D\bar{\psi} D\psi \exp[- \int d^3x F_{\mu\nu}^2 + \int d^3x \bar{\psi} D[A] \psi], \quad (35)$$

where $D[A]$ is the covariant Dirac operator for (massive) fermions. It is important to stress that the gauge field integration in (34) extends *over all gauge field configurations* (See any Quantum Field Theory textbook). Usually, one makes this explicit by means of the Faddeev-Popov technique ending with:

$$Z = \int \Delta_{FP}[A] DA_\mu \delta(F[A^\omega]) D\omega D\bar{\psi} \psi \exp[- \int d^3x F_{\mu\nu}^2 + \int d^3x \bar{\psi} D[A] \psi] \quad (36)$$

Here $F[A^\sigma] = 0$ is the gauge fixing condition selecting one representative A^σ over each gauge orbit and $\Delta_{FP}[A]$ is the corresponding Faddeev-Popov determinant related to the natural metric over orbit space $\Gamma[A^\sigma]$ and the scale $\rho[A^\sigma]$ of each orbit [14]:

$$\Delta_{FP}[A^\sigma] \equiv \rho[A^\sigma] (\Gamma[A^\sigma])^{\frac{1}{2}}. \quad (37)$$

Finally, $D\omega$ is the volume element on the group of gauge transformations.

Usually one eliminates the ω -dependence in the integrand in (36) by changing variables:

$$A \rightarrow A' = A^{-\omega} \quad (38)$$

$$\psi \rightarrow \psi' = \exp[i\omega] \psi \quad (39)$$

$$\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} \exp[-i\omega] \quad (40)$$

If the associated Jacobian is trivial, itegration over ω factorizes. The point is that in a monopole background, the fermionic measure changes non-trivially under transformations (42):

$$D\mu_A[\psi] \equiv \exp(-S_F[A, \bar{\psi}, \psi]) D\bar{\psi} D\psi = J[A, \omega] D\mu_A[\psi'] \quad (41)$$

with

$$J[A, \omega] = \frac{\det D[A^\omega]}{\det D[A]} \quad (42)$$

The reason why $J[A, \omega] \neq 1$ is the following: each one of the determinants in (42) contains, as we have seen, a Chern-Simons term which is not invariant under gauge transformations in a monopole background. Indeed:

$$S_{CS}[A^\omega] = S_{CS}[A] + \frac{e^2}{8\pi^2} \int d^3x B_\mu^{mon} \partial^\mu \omega \quad (43)$$

Integrating by parts and dropping surface terms one has:

$$S_{CS}[A^\omega] - S_{CS}[A] = -\frac{e^2}{8\pi^2} \int d^3x \partial^\mu B_\mu^{mon} \omega \quad (44)$$

Now, in this abelian example, the magnetic field of a monopole at $\vec{x} = \vec{R}$ with magnetic charge n satisfies:

$$\partial^\mu B_\mu^{mon} = \frac{4\pi n}{e} \delta^3(\vec{x} - \vec{R}) \quad (45)$$

(Subtleties related to the construction of abelian monopoles, in particular concerning the appearance of Dirac strings will not be taken into account since we have in mind the $SO(3)$ model where regular monopole solutions without Dirac strings exists. As we shall explain, the arguments presented in the abelian case are completely rigorous in the non-abelian one).

Then, from eqs.(43)-(44) we have:

$$S_{CS}[A^\omega] - S_{CS}[A] = \frac{1}{2\pi} n \omega(\vec{R}) \quad (46)$$

With this, the Jacobian (42) becomes:

$$J[A, \omega] = \exp[-in\omega(\vec{R})] \quad (47)$$

so that, when one integrates out ω all topologically non-trivial (i.e. $n \neq 0$) sectors are wiped out from the partition function since:

$$\int \prod_x d\omega(x) \exp[-in\omega(\vec{R})] \propto \delta_{n,0} \quad (48)$$

Even configurations with total net charge zero but consisting of a superposition of equal number of monopoles and antimonopoles do not contribute. As an example, consider a superposition of a $+1$ monopole located at \vec{R}_1 and an antimonopole of charge -1 at \vec{R}_2 . Then, instead of (48) one has for the ω -integration:

$$\begin{aligned} \int \prod_x d\omega(x) \exp[-i\omega(\vec{R}_1) + i\omega(\vec{R}_2)] &= \int d\omega(\vec{R}_1) \exp[-i\omega(\vec{R}_1)] \times \\ &\int d\omega(\vec{R}_2) \exp[i\omega(\vec{R}_2)] = 0 \end{aligned} \quad (49)$$

Thus we see that the partition function only picks a contribution from the no-monopole sector. Since confinement was precisely produced by monopole contributions, we see that when fermions (or a Chern-Simons term) is included, electric charges are no more confined by a linear potential. The only modification to the arguments above in the non-Abelian case arises from the fact ω takes values in the Lie algebra of the gauge group. For monopoles such that the residual $U(1)$ symmetry corresponds to the 3^{rd} $SO(3)$ direction, this leads to a Jacobian of the form:

$$J[A, \omega] = \exp[-in\omega^3(\vec{R})] \quad (50)$$

From this results, the conclusions reached for the abelian case trivially extend to the non-abelian model.

5 A two-dimensional model

As we stated in the Introduction, in any model where classical invariances might be spoiled at the quantum level one should revise instanton calculations since topologically non-trivial sectors might be wiped out from the correctly gauge-fixed partition function. In the precedent Section we showed how parity anomaly, through the emergence of a Chern-Simons term when

computing the fermionic path-integral, eliminates monopole contributions so that confinement of electric charge is destroyed.

There is a natural candidate to analyse whether the same phenomenon happens: the 2-dimensional abelian Higgs model coupled to *chiral* fermions. As it is well-known, the abelian Higgs model, with action:

$$S_H = \int d^2x \left(-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |(\partial_\mu - ieA_\mu)\phi|^2 + V[|\phi|^2] \right) \quad (51)$$

has vortex solutions [5] which can be taken as instantons in 2-dimensional Euclidean space. (Here, ϕ is a complex scalar and $V[|\phi|^2]$ a symmetry breaking potential having its minimum at $\phi = \phi_0$). Precisely when one takes into account this instantons in a non-perturbative analysis of the model, one discovers screening of (fractional) electric charge due to Debye screening produced in the vortex plasma [15],[16],[17]. More recently, the model has received much attention since it provides a laboratory to analyse if instanton effects can lead at high energy to fermion number violation, a phenomenon of main relevance in the analysis of the Standard Model [18]-[19].

What happens if one adds chiral (say left-handed) fermions to the model? Of course, the corresponding fermionic current is not conserved due to the presence of the anomaly. Nevertheless, it is by now accepted that the so-called anomalous models can be consistently quantized if one correctly takes into account the gauge degrees of freedom [20],[21],[22],[23]. Of course many questions about renormalizability and unitarity of the resulting quantum theory remain to be investigated but in the particular case of 2-dimensional models, this problems do not exists [20] so that it is a sensible question to analyse whether integration over gauge degrees of freedom wipes out instanton effects as it does in the 3-dimensional model described in Section 4.

To answer this question, let us add to the abelian Higgs action (51) left-handed fermions with action:

$$S_F = \int d^2x \bar{\psi} D[A] \psi \quad (52)$$

$$D[A] = (i \not{\partial} + e \not{A}) \frac{(1 + \gamma_5)}{2} \quad (53)$$

and consider the partition function:

$$Z = \int D\phi D A_\mu D \bar{\psi} D \psi \exp[-(S_H + S_F)] \quad (54)$$

As before, we write the A_μ measure à la Faddeev-Popov, ending up with:

$$Z = \int D\phi DA_\mu \Delta[A] \delta(F[A]) J[A, \omega] D\omega \exp[-(S_H + S_F)]. \quad (55)$$

Again, we have to determine whether the Jacobian $J[A, \omega]$,

$$J[A, \omega] = \frac{\det D[A^\omega]}{\det D[A]} \quad (56)$$

is trivial or not. To see this, let us note that each determinant in (56) is not defined since the Dirac operator (52) does not have an eigenvalue problem (it maps negative chirality fermions into positive chirality ones). The chirality-flip problem is usually overcome by defining:

$$\det D[A] \equiv \det \hat{D}[A]|_{reg}, \quad (57)$$

where

$$\hat{D}[A] = D[A] + i \not{\partial} \frac{1}{2} (1 + \gamma_5) \quad (58)$$

and $|_{reg}$ means that an appropriate regularization scheme has been adopted to make sense from the (originally unbounded) product of eigenvalues defining the determinant. Now, the addition of free right-handed fermions solves the chirality-flip problem but creates a new one: under a gauge transformation

$$A_\mu \rightarrow A_\mu^\theta = A_\mu + \frac{1}{e} \partial_\mu \theta, \quad (59)$$

the Dirac operator $\hat{D}[A]$ does not transform as a covariant derivative and then one has in general

$$\det \hat{D}[A^\theta]|_{reg} \neq \det \hat{D}[A]|_{reg}. \quad (60)$$

This means that $J[A, \omega]$ is, in principle, non-trivial. In fact, it is easy to find that [22]-[23]:

$$\log J[A, \omega] = -\frac{1}{4\pi} \int d^2x \left[\frac{(a-1)}{2} \partial_\mu \omega \partial_\mu \omega + e(a-1) A_\mu \partial_\mu \omega + e \epsilon_{\mu\nu} A_\mu \partial_\nu \omega \right]. \quad (61)$$

Here a is a real parameter which takes into account regularization ambiguities which arise when computing gauge non-invariant determinants [20]. This

non-trivial Jacobian induces a Wess-Zumino term which absorbs the anomaly rendering the theory gauge-invariant. Indeed, if we define the fermionic effective action S_{eff} :

$$\exp(-S_{eff}[A]) \equiv \int D\omega D\bar{\psi} D\psi J[A, \omega] \exp(-S_F[A, \bar{\psi}, \psi]), \quad (62)$$

one can easily verify, using the one-cocycle [12] condition satisfied by the Jacobian $J[A, \omega]$,

$$\log J[A, \theta + \omega] = \log J[A, \theta] + \log J[A^\theta, \omega], \quad (63)$$

that S_{eff} is gauge-invariant:

$$S_{eff}[A^\theta] = S_{eff}[A]. \quad (64)$$

This result implies that the fermionic current, defined as $\delta S_{eff}/\delta A_\mu$ is conserved. We shall then take as the partition function for the chiral Abelian Higgs model:

$$Z = \int D\phi D A_\mu \Delta[A] \delta(F[A]) J[A, \omega] D\omega \exp[-(S_H[A, \phi] + S_{eff}[A])] \quad (65)$$

As we stated above, eq.(64) guarantees conservation of the fermionic current and this indicates that the theory with partition function Z given by eq.(65) should be consistently quantized. Of course, in general, unitarity and renormalizability must be investigated. For the two-dimensional chiral Schwinger model and its non-abelian extension [24], it has been shown (see [25] and references therein) that the proposal of refs.[22]-[23] leads to a consistent, unitary and Lorentz invariant quantum theory both in the path-integral and canonical quantization approaches [26]. As we shall see below, the same holds for the chiral Higgs model in two dimensions.

What about instanton contributions to the theory with partition function (65) ? As we stated above, the Abelian Higgs model has vortex-like solutions [5] which can be taken as instantons in the computation of non-perturbative effects [15]-[17]. Asymptotically, a Nielsen-Olesen vortex configuration takes the form:

$$\lim_{r \rightarrow \infty} A_\mu^{vortex}(r, \varphi) = \frac{n}{e} \partial_\mu \varphi \quad (66)$$

$$\lim_{r \rightarrow \infty} \phi^{vortex}(r, \varphi) = \exp(in\varphi)\phi_0, \quad (67)$$

Such a configuration carries n units of magnetic flux (i.e., it has a topological charge equal to n):

$$\frac{e}{2\pi} \oint A_\mu^{(vortex)} dx^\mu = n. \quad (68)$$

The path-integral can then be performed in each topological sector (A configuration satisfying (66)-(67) being a representative in each sector). Using a superscript " n " to indicate the topological sector to which Higgs and gauge fields belong, the partition function will then be written in the form:

$$Z = \sum_n \int D\phi^{(n)} DA_\mu^{(n)} \Delta[A^{(n)}] \delta(F[A^{(n)}]) \exp(-S_{Higgs}[\phi^{(n)}, A^{(n)}] - S_{eff}[A^{(n)}]). \quad (69)$$

A comment on zero-modes of the Dirac operator and the fermionic determinant appearing in the effective action (62) is here in order. We know that the operator $\mathcal{D}[A^{(n)}]$ acting on Dirac fermions has $|n|$ square integrable zero-modes. For $n > 0$ ($n < 0$) these zero-modes are right handed (left-handed) [28]. Hence, the corresponding regularized fermion determinant vanishes for all $n \neq 0$. This automatically ensures that only the $n = 0$ sector contributes to Z for Dirac fermions [29]. On the contrary, the operator $\hat{D}[A^{(n)}]$ has only left-handed zero-modes since in the right-handed sector it coincides with the free Dirac operator which does not have normalizable zero modes). Then $\det \hat{D}[A^{(n)}]$ vanishes only in the case $n < 0$. Consequently Z reduces to:

$$Z = \sum_{n>0} \int D\phi^{(n)} DA_\mu^{(n)} \Delta[A^{(n)}] \delta(F[A^{(n)}]) \exp(-S_H[\phi^{(n)}, A^{(n)}] - S_{eff}[A^{(n)}]). \quad (70)$$

We have now to use the explicit result (61) for the Jacobian. Of course, since we are working in topologically non-trivial sectors, we must not drop surface terms when performing the ω -integral in S_{eff} . Indeed, gauge parameters ω not vanishing at infinity are compatible with any imposed boundary condition at $r = \infty$ since for example $\omega = 2\pi$ is equivalent to $\omega = 0$. This gives, for the second term in the argument of the exponential in (69):

$$e(a-1) \int d^x A_\mu^{(n)} \partial_\mu \omega = e(a-1) \int d^2 x \omega \partial_\mu A_\mu^{(n)} + e(a-1) \oint dx^\mu \omega A_\mu^{(n)} \quad (71)$$

or

$$e(a-1) \int d^x A_\mu^{(n)} \partial_\mu \omega = e(a-1) \int d^2 x \omega \partial_\mu A_\mu^{(n)} + e(a-1) 2\pi n \omega_\infty \quad (72)$$

where we have used (68) and called ω_∞ the value of ω at infinity. We then see that for $n \neq 0$ surface terms do contribute in a non-trivial way. With all this, we get after integrating over ω :

$$\begin{aligned} \exp(-S_{eff}[A^{(n)}]) &= \int D\bar{\psi} D\psi \exp[-(S_F[A^{(n)}, \bar{\psi}, \psi] + \\ &\quad + \frac{e^2}{8\pi(a-1)} \int d^2 x A_\mu A_\mu + F^{(n)})]. \end{aligned} \quad (73)$$

The second term in the argument of the exponential is the usual one obtained (in the Lorentz gauge) after integration over ω when non-trivial topological sectors are not taken into account [22]-[23]. The third term precisely corresponds to the border contribution and is given by:

$$F^{(n)} = \mathcal{N} \lim_{R \rightarrow \infty} F^{(n)}(R), \quad (74)$$

where

$$F^{(n)}(R) = \frac{n^2 a^2}{8(a-1)} \log R, \quad (75)$$

and \mathcal{N} is a constant.

Then, after taking the limit $R \rightarrow \infty$ we have:

$$\exp(-S_{eff}[A^{(n)}]) = \exp(-S_{eff}[A^{(0)}]) \delta_{n,0} \quad (76)$$

and hence the partition function Z (eq.70) only picks contribution from the $n = 0$ sector:

$$Z = \int D\phi^{(0)} D A_\mu^{(0)} \Delta[A^{(0)}] \delta(F[A^{(0)}]) \exp(-S_H[\phi^{(0)}, A^{(0)}] - S_{eff}[A^{(0)}]). \quad (77)$$

As announced, the chiral Higgs model, though anomalous can then be consistently quantized and only the $n = 0$ sector contributes to the partition function. The value of $S_{eff}[A]$ can be easily evaluated from (62). The answer (in the Lorentz gauge) is [22]-[23]:

$$S_{eff}[A] = \frac{a^2}{8\pi(a-1)} \int d^2x A_\mu A_\mu. \quad (78)$$

Inserting this value in (77) we see that the result coincides with that corresponding to the Abelian Higgs model with *Dirac* fermions except for the fact that the vector meson mass m_v is given by

$$m_v^2 = e^2(|\phi_0|^2 + a^2/4\pi(a-1)). \quad (79)$$

Then, Z in (77) defines a unitary, positive model for any value of the parameter a such that $m_v^2 > 0$. The existence of a whole range of the undetermined parameter a for which the model is consistent, Lorentz invariant and unitary with a vector meson mass which is not fixed by gauge invariance (as it happens for models with Dirac fermions) is typical of anomalous gauge theories at least in $d = 2$ dimensions. One can think that Z defines a family of quantum theories and that the ultimate value for a should be determined by physical considerations (see ref.[30] for a discussion on these facts).

In summary, we have shown in this Section that the chiral Abelian Higgs model in two dimensions, though anomalous, can be consistently quantized following the proposal of refs.[22]-[23]. The anomaly is cancelled by a "Wess-Zumino" term, as suggested in [12] and as a result, instanton contributions are eliminated from the partition function defining the quantum theory. The procedure can be straightforwardly generalized to non-Abelian models and one can also envisage the analysis of four dimensional Higgs models. However, in this last case the issues of unitarity and renormalizability should be carefully investigated.

6 Questions

N.Bralic: *You used an abelian example to show that integrating over all field configurations eliminates monopole contributions to the partition function. In your proof, it was crucial that $\partial_\mu B_\mu = \delta^3(\vec{x} - \vec{R})$. Can you explain how*

does your argument work for the non-abelian monopole, for which you do not have such a relation?

Answer: The 't Hooft-Polyakov (charge 1) monopole solution reads [1]-[2]:

$$\phi^a = \hat{x}^a f(r) \quad (80)$$

$$A_\mu^a = \epsilon_{a\mu i} \hat{x}^i \frac{(1 - K(r))}{r} \quad (81)$$

with $\hat{x}^a \equiv x^a/r$, $f(0) = 0$, $f(\infty) = \phi_0$, $K(0) = 1$, $K(\infty) = 0$. The corresponding magnetic field is (see eq.(5)):

$$B_\mu = \frac{\hat{x}_\mu}{r^2} (1 - K^2) \quad (82)$$

so that:

$$\Phi = \int B_\mu dS_\mu = 4\pi \quad (83)$$

Let us consider a family of gauge transformations g of the form:

$$g = \exp\left[\frac{i}{2}\omega(r)\sigma^a\hat{x}^a\right] \quad (84)$$

such that $\omega(0) = 0$ (in order to avoid singularities) and $\omega(\infty) = \omega$ (with ω a non-zero constant). Under such a transformation, the monopole configuration (81) becomes:

$$\left(A_\mu^a\right)^g = \epsilon_{a\mu i} \hat{x}^i \frac{(1 - K \cos \omega(r))}{r} + (\delta_{a\mu} - \hat{x}_\mu \hat{x}_a) \frac{K \sin \omega(r)}{r} + \hat{x}_\mu \hat{x}_a \frac{d\omega(r)}{dr} \quad (85)$$

so that asymptotically one has:

$$\left(A_\mu^a\right)^g = \epsilon_{a\mu i} \frac{\hat{x}^i}{r} + \hat{x}_\mu \hat{x}_a \frac{d\omega(r)}{dr} \quad (86)$$

Let us consider how the Chern-Simons action changes under such a class of gauge transformations:

$$\delta S_{CS}[A] = \frac{2}{e^2} \epsilon_{\mu\nu\alpha} \text{tr} \left[\frac{1}{3} \int d^3x (\partial_\mu g g^{-1})(\partial_\nu g g^{-1})(\partial_\alpha g g^{-1}) + \int dS_\mu A_\nu \partial_\alpha g g^{-1} \right] \quad (87)$$

or

$$\delta S_{CS}[A] = \frac{16\pi^2}{e^2} n[g] + \frac{2}{e^2} \epsilon_{\mu\nu\alpha} \text{tr} \int dS_\mu A_\nu \partial_\alpha g g^{-1} \quad (88)$$

with $n[g]$ the winding number of the g -transformation (note that with $\mu = \frac{e^2}{8\pi}$ as in eq.(28) $\exp(-S_{CS})$ is not affected by the first term in the r.h.s. of eq.(88)). Let us consider the second term in the r.h.s. of eq.(28). An explicit calculation using the form (85) for the monopole configuration gives:

$$\delta S_{CS} = \frac{16\pi^2}{e^2} n[g] + \frac{2}{e^2} \int d^3x \frac{1}{r^2} \frac{d\omega}{dr} \quad (89)$$

or

$$\delta S_{CS} = \frac{16\pi^2}{e^2} n[g] + \frac{8\pi}{e^2} \omega \quad (90)$$

Now, when integrating out $\omega(x)$ in (36) one has to include this family of gauge transformations with $\omega \in [0, 2\pi]$, so that there is an integral of the form:

$$\int_0^{2\pi} d\omega \exp(i\omega) = 0 \quad (91)$$

which wipes out the charge-1 monopole contribution from Z . Similar arguments hold for charge- n sectors whenever $n \neq 0$.

H.Banerjee: *You stated that the Chern-Simons action arises as a one-loop effect when computing the (3-dimensional) fermion determinant. This result crucially depends on the regularization scheme you adopt. You may obtain the Chern-Simons term using Pauli-Villars method but other prescriptions do not give parity-violating results, in particular when the fermion mass is strictly zero.*

Answer: Although originally the Chern-Simons term was obtained using the Pauli-Villars method for regularizing the fermion determinant, one can adopt

alternative methods and still obtain the same result. In particular, in Ref.[31] we have employed the well-honored ζ -function method showing that a careful application of Seeley's technique [32] **does lead to a Chern-Simons term even when the fermion mass is strictly zero**, thus contradicting the results in [33]. Even the sign ambiguity in front of the CS action is reobtained using ζ -function as a result of the choice of upper or lower half-plane when computing the finite part of the $K_{-1}(x, x, D)$ kernel without the necessity of introducing a fermion mass (see [31]).

C.Teitelboim:it There are many young people in the audience and we are morally responsible for them. You have discussed an anomalous gauge theory as if it could be consistently quantized but it should be stressed that anomalous theories are not unitary and hence inconsistent. Can you comment on this?

Answer: The model I have discussed (the Abelian chiral Higgs model) is a **two-dimensional** model. As in the chiral Schwinger Model case [20], one can show that it can be consistently quantized in a unitary and Lorentz invariant way [13]. Of course the issue of unitarity and renormalizability in **four-dimensional** anomalous gauge theories using the approach described in Section 5 is not clear and as you said it touches moral aspects which I prefer not to discuss.

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